Macroeconomic Theory II, Spring 2014: Homework 5

Problem 1: Bond and stock

Consider an economy populated by a continuum of measure one of infinitely lived, ex-ante equal agents with preferences over sequences of consumption given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}\right).$$

Agents face individual uninsurable endowment shocks ε_{it} to labor productivity, where $\varepsilon_{it} \in \{\varepsilon_1, ..., \varepsilon_N\}$. This process is governed by the Markov chain $\pi_\varepsilon\left(\varepsilon', \varepsilon\right) = \Pr\left\{\varepsilon_{i,t+1} = \varepsilon' | \varepsilon_{it} = \varepsilon\right\}$. Production takes place through the aggregate technology $Y_t = z_t K_t^{\alpha} H_t^{1-\alpha}$ where K_t and H_t are, respectively, aggregate capital and aggregate efficiency units of labor at time t. Capital depreciates at rate δ. Aggregate total factor productivity $z_t \in \{z_b, z_g\}$ follows a Markov process $\pi_z\left(z', z\right) = \Pr\left\{z_{t+1} = z' | z_t = z\right\}$.

Agents can hold two assets: risky capital a_{it} and a risk-free one-period bond b_{it}. Let q_t be the price of the bond at time t: buying b units of the risk-free bond at price q_t at time t guarantees b units of final good next period. Assume that agents face borrowing limits $\left(-\bar{a}, -\bar{b}\right)$ on the two assets. Labor and asset markets are competitive.

a) Write down the problem of the household in recursive form, making explicit the individual and the aggregate state variables.

b) Define a recursive competitive equilibrium for this economy.

c) Outline a numerical algorithm that solves for the equilibrium allocations and prices.

Problem 2: Moody Government

Consider an economy populated by a continuum of measure one of infinitely lived, ex-ante equal agents with preferences over sequences of consumption and leisure given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}, 1 - h_{it}\right).$$

Agents face individual uninsurable endowment shocks ε_{it} to labor productivity, where $\varepsilon_{it} \in \{\varepsilon_1, ..., \varepsilon_N\}$. This process is governed by the Markov chain $\pi\left(\varepsilon', \varepsilon\right) = \Pr\left\{\varepsilon_{i,t+1} = \varepsilon' | \varepsilon_{it} = \varepsilon\right\}$. Agents can save and borrow through a unique *non*-state-contingent asset a_{it}. Assume that they can borrow up to the natural limit (what is it?).

Agents supply h_{it} hours of labor to the market, where each hour has productive efficiency equal to ε_{it}. Labor and asset markets are competitive and clear, every period, with prices w_t (per efficiency unit) and r_t, respectively.

Production takes place through the aggregate technology $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$ where K_t and L_t are, respectively, aggregate capital and aggregate units of labor in efficiency units at time t.

The government in this economy sets government expenditures G_t according to the mood of its president, to either G_h or G_l, with $G_h > G_l$. The mood of the president follows a publicly known random process $\phi(G', G) = \Pr\{G_{t+1} = G' | G_t = G\}$. Proportional taxes on consumption expenditures τ_t^c adjust to balance the budget every period.

a) Write down the problem of the household in recursive form, making explicit the individual and the aggregate state variables.

b) Define a recursive competitive equilibrium for this economy.

c) Outline a numerical algorithm that solves for the equilibrium allocations and prices.

Suppose you want to compute the welfare costs of living under a moody government, compared to facing a government with a stable mood (and stable expenditures $\bar{G}$, for all t).

d) Define your preferred welfare criterion and explain how you would calculate this welfare loss, in practice.

Problem 3: Welfare Cost of Business Cycles

Consider the Krusell-Smith model that we studied in class (the one described in the lecture notes, i.e. the neoclassical growth model with idiosyncratic uninsurable risk, aggregate risk and inelastic labor supply).

a) Describe how you would calculate the welfare gains from eliminating business cycles (i.e. eliminating fluctuations in z) for an agent in a generic state (a, ε). Think, in particularly, about how to handle the fact that the evolution of idiosyncratic uncertainty ε [the risk you do not eliminate] depends on aggregate uncertainty z [the risk you do eliminate] through the joint Markov chain $\pi(z', \varepsilon', z, \varepsilon)$.