Macroeconomic Theory II, Spring 2014: Homework 4

Problem 1

Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy is populated by a continuum of measure one of infinitely-lived households indexed by i on $[0, 1]$. Preferences for individual i are given by

$$U\left(c_0^i, c_1^i, ...\right) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right),$$

where $\beta \in (0, 1)$ is the discount factor, $c_t \geq 0$ denotes consumption at time t, $u' > 0$ and $u'' < 0$. Each household i in period t receives a stochastic idiosyncratic endowment of the consumption good $\varepsilon_t^i \in \{\varepsilon_h, \varepsilon_l\}$ which follows a Markov chain where $\pi_{hh}^{\varepsilon} = \Pr\left\{\varepsilon_t^i = \varepsilon_h | \varepsilon_{t-1}^i = \varepsilon_h\right\}$, $\pi_{ll}^{\varepsilon} = \Pr\left\{\varepsilon_t^i = \varepsilon_l | \varepsilon_{t-1}^i = \varepsilon_l\right\}$, $\pi_{hl}^{\varepsilon} = 1 - \pi_{hh}^{\varepsilon}$, and $\pi_{lh}^{\varepsilon} = 1 - \pi_{ll}^{\varepsilon}$. Households can trade a risk-free asset a in zero-net supply with return R_t, and they can accumulate debt up to the exogenous borrowing limit $-\bar{b}$.

a) Define a stationary recursive competitive equilibrium (RCE) for this economy, and discuss its existence and uniqueness.

Suppose now that $\bar{b} = 0$, and that $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

b) What is so special about this case? Determine the equilibrium risk-free return as a function of the model's parameters and prove that the return is lower than $1/\beta$. Explain how the risk-free rate changes with the ratio $\varepsilon_h/\varepsilon_l$, with the persistence parameter π_{hh}, and with risk aversion γ.

Problem 2

Consider a stationary economy populated by a continuum of measure one of infinitely lived, ex-ante equal agents with preferences over sequences of consumption and leisure given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_{it}\right) + e^{\psi_{it}} v\left(1 - h_{it}\right)\right].$$

Agents face individual shocks to their preference for leisure ψ_{it}, which follow the stochastic process $\psi_{it} = \rho\psi_{i,t-1} + \eta_{it}$, with $\eta_{it} \sim N\left(0, \sigma_\eta\right)$. Agents can save but cannot borrow. Production takes place through the aggregate technology

$$C_t + K_{t+1} - (1 - \delta) K_t = AK_t^{\alpha} H_t^{1-\alpha}$$

where C_t, K_t and H_t are, respectively, aggregate consumption, aggregate capital, and aggregate hours at time t. Labor and asset markets are competitive and clear, every period, with prices w_t and r_t, respectively.

The government taxes capital income at a fixed flat rate τ. Tax revenues are returned to agents as tax-exempt lump-sum transfers b. Households can evade taxes by choosing every period t the fraction of capital income ϕ_{it} to declare in their tax return, i.e., the fraction of capital income on which they pay taxes. Let x_{it} be the total undeclared taxes at time t.

The government, knowing that agents may have evaded taxes at time $t-1$, at time t can monitor and perfectly verify the past period individual tax returns. Let π be the probability that, at time t, the time $t-1$ tax return of a household is subject to monitoring. The household finds out whether her $t-1$ period tax return is monitored at the beginning of period t, i.e., before consumption decisions are taken. In the event the household is caught, at time t the tax agency gives her a fine equal to $z(x_{i,t-1})$, where $x_{i,t-1}$ is the tax amount due from the past period, with $z(0) = 0$ and $z(x_{i,t-1}) > x_{i,t-1}$.

a) Write down the problem of the household in recursive form, making explicit the individual and the aggregate state variables.

b) Write down the individual first-order necessary condition that characterizes the optimal tax evasion choice.

c) Define a stationary recursive competitive equilibrium for this economy.

Problem 3

Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy is populated by a continuum of measure one of infinitely-lived households indexed by i on $[0,1]$. Preferences for individual i are given by

$$U\left(c_0^i, c_1^i, ...; h_0^i, h_1^i, ...\right) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i, 1 - h_t^i\right),$$

where $\beta \in (0,1)$ is the time discount factor, $c \geq 0$ denotes consumption and $h \geq 0$ hours worked. Household i at time t supplies the optimal amount of hours worked h_t^i to a competitive labor market, taking the hourly wage ω_t as given.

Each household owns a single private firm, i.e. firm i is identified with household i. Each firm produces the final good y (which can be used for conumption or investment) through

$$y_t^i = F\left(z_t^i, k_t^i, n_t^i\right),$$

where F is a production technology displaying constant-returns in capital and labor. The variable z_t^i represents a firm-specific productivity shock which follows the Markov process $\pi(z', z) = \Pr\left\{z_{t+1}^i = z' | z_t^i = z\right\}$ with $z_{\min} = 0$.

Households can freely hire labor on the market for their firm, but here is no market to trade claims on physical capital, thus households can only invest the capital they own in their firm. Moreover, $k_t^i \geq 0$. Capital depreciates at rate δ. Let π_t^i denote the profits accruing to individual i from operating her firm at time t.

Households can trade a risk-free bond b in zero-net supply with return R_t and they can accumulate debt up to the "natural borrowing constraint".

Assume that the economy is in a stationary equilibrium.

a) What is the natural borrowing constraint faced by each household?

b) List explicitly the individual state variables and write down the problem of the household in recursive formulation.

c) Define a Recursive Competitive Equilibrium (RCE) for this economy.

d) Discuss the existence and uniqueness of the RCE for this economy.

Imagine now that a new financial market opens allowing agents to invest their own k in other households' firms.

e) What is a simple way to represent the new economy with this additional market? In particular, list what the individual state variables become in this case.