Macroeconomic Theory II, Spring 2014: Homework 1

The assignment is due next Thursday.

Problem 1

Consider a two-sector version of the neoclassical growth model, where one sector produces the consumption good c_t with technology

$$c_t = z_t^c f\left(k_t^c, n_t^c\right)$$

and the other sector produces the investment good i_t with technology

$$i_t = z_t^i f\left(k_t^i, n_t^i\right).$$

Assume f is CRS, strictly concave and increasing in both argument. Note that f is the same in both sectors.

Aggregate capital is $k_t = k_t^c + k_t^i$ and accumulates according to the law of motion $k_{t+1} = (1-\delta) k_t + i_t$. Capital used in the consumption sector is k_t^c and capital used in the investment sector is k_t^i. Aggregate labor input is $n_t = n_t^c + n_t^i$, where n_t^c is labor supplied in the consumption sector and n_t^i is labor supplied in the investment sector.

The representative household has period utility $u(c_t, 1-n_t)$ and preferences are time-separable with discount factor β. All markets are competitive and both labor and capital are free to move across sectors. Let p_t^c be the price of the consumption good and p_t^i be the price of the investment good.

1) Show that this economy aggregates into a one-sector growth model, i.e. the production structure can be summarized by an aggregate production technology $\tilde{z} f(k_t, n_t)$. What is the interpretation of the ratio (p_t^c / p_t^i) in equilibrium?

Problem 2

Consider a version of the neoclassical growth model where the economy is inhabited by two types of agents $i = 1, 2$ with measure μ_i where $\mu_1 + \mu_2 = 1$. Agents of type i solve

$$\max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t \frac{\left(c_{it}^{\alpha}\left(1-h_{it}\right)^{1-\alpha}\right)^{1-\gamma}}{1-\gamma}$$

$$s.t.$$

$$c_{it} + a_{i,t+1} = a_{i,t}\left[1 + r_t\left(1-\tau_t\right)\right] + w_t \varepsilon_i h_{it} + T^0,$$

$$a_{i0} \text{ given for } i = 1, 2$$

where ε_i are efficiency units of labor endowed to type i, τ_t is a capital-income tax rate and T^0 are lump-sum transfers. The representative firm produces with CRS technology $F(K_t, N_t)$ where N_t is aggregate labor input in efficiency units. The government budget constraint is balanced in every period, i.e.,

$$T^0 = \tau_t K_t r_t$$

Finally, the aggregate resource constraint is

$$C_t + K_{t+1} = F(K_t, N_t) + (1 - \delta) K_t.$$

1) Show that in steady-state the distribution of wealth is indeterminate.

2) Does this economy admit a representative agent formulation?

Suppose that the economy is, initially, in a particular steady-state with transfers T^0 and imagine that the government raises transfers to $T^1 > T^0$.

3) Does the new set of steady-states with higher transfers display higher or lower capital stock? Draw the equilibrium dynamics of this economy between steady-states in the space of wealth holding (a_1, a_2) for the two agents. Is the final steady-state uniquely determined? How would you compute the dynamics?

Problem 3

Consider a pure exchange economy, where time is discrete, indexed by $t = 0, 1, 2, ...$ and continues forever. The economy is populated by 2 individuals $i = 1, 2$ with logarithmic period utility $\ln(c_t^i)$ and discount factor equal to $\beta \in (0, 1)$ who trade a non-storable consumption good c_t. Agents have deterministic endowment streams $\{e_t^i\}_{t=0}^{\infty}$ of the consumption good given by

$$e_t^i = \begin{cases} 0 \quad if \;\; (t+i) \text{ is even} \\ 2 \quad if \;\; (t+i) \text{ is odd} \end{cases}$$

Agents behave competitively. All markets open at time zero and contracts are traded specifying how many units of consumption good will be exchanged at each time t between the two agents.

1) Define an Arrow-Debreu competitive equilibrium and verify that the first Welfare Theorem holds.

2) Solve for the Arrow-Debreu equilibrium, i.e., characterize the equilibrium sequences of prices and allocations of consumption good among agents.

3) Define a Pareto optimal allocation for this economy. Write down the Negishi Planner's problem where the Planner gives weight α to agent 1 and $(1-\alpha)$ to agent 2. Solve the Planner's problem for arbitrary weights $\alpha \in (0,1)$.

4) Characterize the transfer function that allows to map any Pareto optimum, for given weights $(\alpha, 1-\alpha)$, into a competitive equilibrium with transfers among agents and determine the specific value for the planner weights that select exactly the competitive equilibrium you have computed in point 2).

Problem 4

Prove that if (and only if) the indirect utility function has the form $v_i(p, a_i) = \alpha_i(p) + \beta(p)a$, for an agent with utility function u_i, then Engel curves (expenditures as a function of wealth) are linear in wealth .